JOURNAL OF STATISTICAL SCIENCES & DATA ANALYTICS
Department of Statistics, Ahmadu Bello University, Zaria, Nigeria | www.jossda.abu.edu.ng www.jossda.com.ng

A Firefly Optimization Algorithm for Forecasting Based on Time-Variant Fuzzy Time Series

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Abstract

Forecasting has always been a crucial challenge for managers and scientists in terms of making accurate management or investment decisions. Although there are a variety of time series models in the literature, fuzzy time series modelling has attracted attention over the past decade to improve forecast accuracy with a small sample of data. This paper aims to present a fuzzy time series methodology for forecasting exchange rate trends that could serve as a scientific approach toward feasible and sustainable economic development policies. Specifically, we propose a hybrid algorithm to deal with forecasting based on time-variant fuzzy time series and the firefly algorithm, as a highly efficient and recent evolutionary metaheuristic inspired by a firefly and its bioluminescence behaviours. The proposed algorithm determines the length of each interval in the universe of discourse and degree of membership values simultaneously. We calibrate the proposed algorithm on benchmark data in order to compare the results achieved in terms of forecasting accuracy with those of particle swarm optimisation in the literature; then, we apply it to forecast quarterly exchange rates. Numerical results indicate that the proposed algorithm competes well with the particle swarm optimisation approach.
Keywords: Time-variant series; fuzzy time series; firefly algorithm; particle swarm optimisation

1. Introduction

Forecasting has always been a crucial challenge for managers and scientists in terms of making accurate management or investment decisions. In particular, in many manufacturing and services industries, due to insufficient historical data, traditional forecasting methods usually give large margins of error between the predicted and actual values. To solve this problem, Song and Chissom (1993) first proposed the concept of fuzzy time series. The main property of the fuzzy time series is that the values of the demand variables are linguistic values. Despite the variety of time series models in the literature, fuzzy time series modelling has attracted attention over the past decade to improve forecast accuracy under a small sample of data.

In recent years, a vast literature on fuzzy time series has come into play. Fuzzy time series was first introduced by Song and Chissom (1993a). Song and Chissom (1993a) categorised fuzzy time series into two classes: time-variant and time-invariant. It is time-invariant if the relations are only between the current time $\boldsymbol{t}$ and its immediately preceding time $\boldsymbol{t-1}$, otherwise, it is time-variant. Song and Chissom (1993b) proposed an algorithm to analyse time-invariant fuzzy time series. Like other fuzzy inference systems, fuzzy time series consists of three phases: fuzzification, determination of fuzzy relations, and defuzzification. It is commonly understood that all of these phases directly affect the forecasting performance of the method. In the literature, there are many studies in which there is a contribution to each of these phases. In the most preferred approach for the fuzzification phase, the universe of discourse is defined, and it is partitioned into equal or unequal intervals by a proper technique. According to these intervals, fuzzy sets are defined by determining the membership values. Then, observations are mapped into these fuzzy sets by a predefined rule.

Huarng (2001) examined the effect of the length of the interval on forecasting results. Huarng (2001) also suggested two methods which are based on the average and distribution in order to determine the length of the interval. Egrioglu *et al*. (2010, 2011) utilised single-variable constrained optimisation to determine the interval length for first and higher order fuzzy time series models. Huang and Yu (2006a) suggested using a dynamic interval length based on a given proportion. In this approach, the universe of discourse is partitioned by changing interval lengths instead of fixed interval lengths. Some of the next studies have been inspired by the idea of using a dynamic interval length. Yolcu *et al*. (2009) improved the approach proposed by Huarng and Yu (2006a) by using optimisation for determining the best value of the proportion. Davari *et al*. (2009), Kuo *et al*. (2009; 2010), Park *et al*. (2010), and Hsu et al. (2010) determined the dynamic lengths of intervals using particle swarm optimisation (PSO). It was observed that using the PSO method considerably increases forecasting accuracy. Chen and Chung (2006) and Lee *et al*. (2007; 2008) utilised genetic algorithms in the fuzzification phase. Cheng et al. (2008) employed the fuzzy c-means method for fuzzification.

Another important phase that has a significant effect on forecasting performance is the determination of fuzzy relations between observations. For this phase, Song and Chissom (1993b) proposed a method based on fuzzy logic relationships. They used various matrix operations to produce these relationships. Chen (1996) suggested a method based on fuzzy logic group relation tables. The method proposed by Chen (1996) is easier than Song and Chissom's (1993b) method. In the literature, the most preferred way to define fuzzy relations is using fuzzy logic group relation tables. As an alternative to these two methods, Huarng and Yu (2006b) suggested using artificial neural networks to establish fuzzy relations. Aladag et al. (2009; 2010) and Aladag (2012) proposed high-order fuzzy time series forecasting approaches and employed feed-forward neural networks to define fuzzy relations in their methods. Egrioglu et al. (2009a, b, c) suggested bivariate and multivariate models and utilized feed forward neural networks. Alpaslan et al. (2011; 2012) and Yolcu et al. (2013)

Journal of Statistical Sciences & Data Analytics (JOSSDA)
Volume 1, Issue 2, December 2026 | Print ISSN XXXX-XXXX | Online ISSN XXXX-XXXX

DOI: 10.XXXXX/jossda.2026.1.1.015

proposed fuzzy time series approaches using membership values and exploited feed-forward neural networks for determination of fuzzy relations.

In the fuzzification phase of the fuzzy time series approach, there are some common problems such as the decision of the number of intervals, arbitrary determination of interval length, and arbitrary choice of degrees of membership (Egrioglu *et. al*., 2015). In this study, we aim to propose a fuzzy time series forecasting approach using a more recent metaheuristic to improve forecasting accuracy. Specifically, to optimise the length of the interval of discourse and the values of the degree of membership simultaneously.

The rest of the paper is structured as follows. Section 2 describes the methodological framework. Then, the experimental results are presented and discussed in Section 3. Finally, Section 4 provides a conclusion.

2. Materials and Methods

We adopted the procedure in Mahnam and Fatemi Gomi (2012) as a basis to develop the proposed hybrid method. The steps are described as follows:

Step 1: Computation of the variations of the historical data. This can be obtained as $V_t = d_t - d_{t-1}$ $(t = 2,3,\dots,n)$, where the variation V_t of data between time $t(d_t)$ and $t-1(d_{t-1})$.

Step 2: Define the universe of discourse. Find the maximum(D_{max}) and the minimum(D_{min}) among all V_t. The universe of discourse U is then defined as $U = [D_{min} - D_1$ $D_{max} + D_2]$, where D_1 and D_2 are two positive numbers.

Step 3: The firefly algorithm is used to determine the appropriate interval length and degree of membership values. Note: a fuzzy number is considered to fuzzify intervals.

Step 4: Fuzzify the variations of historical data. If the variation is within the scope of u_j, it belongs to fuzzy set A_j. All of the variations must be classified into the corresponding fuzzy sets.

Step 5: Calculate the fuzzy time series $F(t)$ at window base w. The operation matrix $O^w(t)$ and the criterion matrix $C(t)$ are selected to compute the fuzzy forecasted variation $F(t)$.

$$C(t) = F(t-1) = [C_1\ C_2, \cdots, C_m] \tag{1}$$

$$O^w(t) = \begin{bmatrix} F(t-2) \\ F(t-3) \\ \vdots \\ F(t-w) \end{bmatrix} = \begin{bmatrix} o_{11}\ o_{12} \cdots o_{1m} \\ o_{21}\ o_{22} \cdots o_{2m} \\ \vdots \quad \vdots \quad \vdots \quad \vdots \\ o_{(w-1)1}\ o_{(w-2)2} \cdots o_{(w-1)m} \end{bmatrix} \tag{2}$$

Journal of Statistical Sciences & Data Analytics (JOSSDA)
Volume 1, Issue 2, December 2026 | Print ISSN XXXX-XXXX | Online ISSN XXXX-XXXX

DOI: 10.XXXXX/jossda.2026.1.1.015

$$= \begin{bmatrix} R_{11}\ R_{22} \cdots R_{1m} \\ R_{21}\ R_{22} \cdots R_{2m} \\ \vdots \quad \vdots \quad \vdots \quad \vdots \\ R_{(w-1)1}\ R_{(w-1)2} \cdots R_{(w-1)m} \end{bmatrix}$$

where C_j indicates the membership value at the interval u_j within fuzzy set A_j. The fuzzy relation matrix $R(t)$ is computed by performing the following fuzzy operation.

$$R(t) = O^w(t) \otimes C(t) \tag{3}$$

Then $\mathrm{F}(t)$ can be calculated as the maximum of every column in $R(t)$ as follows:

$$\mathrm{F}(t) = \left[max_{k=1,\cdots,w-1\{R_{k1}\}} \cdots max_{k=1,\cdots,w-1}\{R_{km}\}\right] \tag{4}$$

Step 6: Forecasted value. Suppose there are k non-zero values corresponding to intervals. $u_1, u_2, \cdots, u_n$ with their midpoints $m_1, m_2, \cdots, m_k$, respectively. We use the weighted average method to calculate the defuzzified variation.

$$Cv_t = \frac{\sum_{i=1}^{m} f_i m_i}{\sum_{i=1}^{m} f_i} \tag{5}$$

The forecasted value Fv_t at time t is computed as follows:

$$Fv_t = Cv_t + d_{t-1} \tag{6}$$

2.1. The Classical Firefly Algorithm

The firefly algorithm is an evolutionary metaheuristic algorithm inspired by firefly behaviour in nature. Developed by Xin-She Yang in 2007. There are two important issues in FA: The variation of light intensity and the formulation of attractiveness. For simplicity, it is assumed that the attractiveness of a firefly is determined by its brightness, which in turn is associated with the encoded objective function. Each firefly has its own specific attractiveness β, which is judged by other fireflies. In addition, the light intensity also depends on the distance from the source. The light is also absorbed in the media resulting in varying attractiveness and degree of absorption. If a given medium has a fixed light absorption coefficient γ, the light intensity I depends on the value of the distance r between two fireflies.

$$I = I_o e^{-\gamma} \tag{7}$$

Where I_o is the original light intensity

As a firefly's attractiveness is proportional to the light intensity seen by adjacent fireflies, attractiveness β of a firefly is now defined by

Journal of Statistical Sciences & Data Analytics (JOSSDA)
Volume 1, Issue 2, December 2026 | Print ISSN XXXX-XXXX | Online ISSN XXXX-XXXX

DOI: 10.XXXXX/jossda.2026.1.1.015

$$\beta(r_{ij}) = \beta_o e^{-\gamma r^2} \tag{8}$$

Where β_o is the attractiveness when the distance between the two fireflies, represented by r is zero.

The distance between any two fireflies i and j is the Cartesian distance:

$$r_{ij} = \|x_{j-}x_i\| = \sqrt{(x_{i,k} - x_{j,k})^2} \tag{9}$$

The movement of a firefly i is attracted to another brighter firefly j is determined by

$$\boldsymbol{x_i} = x_i + \beta_o e^{-\gamma r^2}(x_j - x_i) + \alpha\epsilon_i \tag{10}$$

It is worth pointing out that eq. (4) is a random walk biased towards the brighter fireflies (Xin-she Yang (2010).

Algorithm 1: Firefly Algorithm

Objective function $f(x)$, $\boldsymbol{x} = (x_1, x_2, \dots, x_d)^T$
Generate initial population of fireflies x_i $(i = 1, 2, \dots, n)$
Light intensity I_i at x_i is determined by $f(x_i)$
Define light absorption coefficient γ
While ($(t < MaxGeneration)$
for $i = 1{:}n$ all n fireflies
  for $j = 1{:}n\ all\ n\ fireflies\ (inner\ loop)$
  if $(I_i < I_j), move\ fireflly\ i\ towards\ j;$ ***end if***
    Vary attractiveness with distance r via $\exp[-\gamma r]$
    Evaluate new solutions and update light intensity
  end for j
end for i
Rank the fireflies and find the current global best **g***
end while
Postprocess results and visualization

Figure 1. Pseudocode for classical FA

Journal of Statistical Sciences & Data Analytics (JOSSDA)
Volume 1, Issue 2, December 2026 | Print ISSN XXXX-XXXX | Online ISSN XXXX-XXXX
 DOI: 10.XXXXX/jossda.2026.1.1.015

2.2. The Proposed Firefly Algorithm

The firefly algorithm (FA) is a population-based metaheuristic algorithm, and it was originally introduced by Xin-She Yang in 2007. It draws inspiration from the flashing behaviour of fireflies when they try to communicate and attract males, translating bioluminescent signals into a powerful optimisation technique. Some of the advantages of the algorithm, compared to other metaheuristics, are simplicity in terms of understanding and implementation, less required computational time, minimal user-defined parameters, and it was also applied successfully for solving many difficult continuous optimisation problems.

In this paper, we propose an integrated approach to determine the length of intervals and membership value simultaneously. In this algorithm, to obtain the best forecasted value, different possible combinations of window base (w) and number of intervals (m) are considered for each combination of (w, m), FA finds a near-optimal solution for the length of intervals and degree of membership values by updating the fireflies based on the MAD objective function and their *pbest* and *gbest* in each iteration.

2.2.1. Algorithm Procedure

The main structure of the FA algorithm is illustrated in Figure 2.

Figure 2. Flowchart for the proposed FA algorithm

2.2.2. Solution representation (Firefly)

The solution generated randomly is coded as a list data structure as follows:

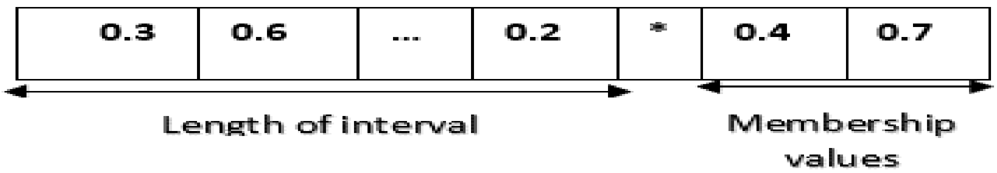

Figure 3. A firefly representing a feasible solution

Journal of Statistical Sciences & Data Analytics (JOSSDA)
Volume 1, Issue 2, December 2026 | Print ISSN XXXX-XXXX | Online ISSN XXXX-XXXX
 DOI: 10.XXXXX/jossda.2026.1.1.015

Figure 3 indicates the simultaneous composition of the interval length of the universe of discourse and its associated membership values. The first part of the list represents the interval length of the universe of discourse, while the second part is the values of membership.

Let the number of intervals be m; a firefly is a list consisting of $m-1$ elements, such that $0 < q_i < 1$, for $i = 1,2,\cdots,m-1$. To determine the intervals, the elements of the corresponding firefly should be multiplied by the universe of discourse, i.e. $b_i = q_i * (D_{max} - D_{min})$. Therefore, the m intervals are as follows:

$$I_1 = (D_{min}, D_{min} + b_1$$

$$I_2 = (D_{min} + b_1, D_{min} + b_1 + b_2)$$

$$\dots$$

$$I_{n-1} = (D_{min} + \sum_{i=1}^{m-2} b_i, D_{min} + \sum_{i=1}^{m-1} b_i)$$

$$I_n = (D_{min} + \sum_{i=1}^{m-1} b_i, D_{max})$$

By updating the position of fireflies in Fig. 2, the elements of the corresponding new list need to be divided by the summation of elements to ensure that each element $\boldsymbol{b_i}$ is in the domain (0, 1). Moreover, the value of memberships should be in the domain (0, 1). Moreover, the second part of the list indicates the membership values of fuzzy intervals. To reduce the solution space of this part of the list, only three intervals can be considered as non-zero membership values. These three intervals are entitled lower, normal, and upper intervals, respectively. Also, the membership values of lower, normal, and upper intervals are bounded between 0.3 and 0.7 as commonly found in the literature, purposely to achieve better results. The important feature of this representation is that all the fireflies generated by the algorithm are feasible solutions, and its application is faster than other representations in the literature.

2.2.3. Neighborhood Structure

The neighbourhood of a firefly is the social environment a firefly encounters. The proposed algorithm uses a social neighbourhood, which is a list of fireflies regardless of their positions. Since it may occasionally happen that the algorithm is easily trapped in local optima, a neighbourhood structure is used to prevent the particle from pre-convergence (Mahnam & SFatemi Ghomi, 2012). Therefore, if after $g1$ iterations, *gbest* shows no improvement, the global neighbourhood will replace the local one. This decision provides the possibility for searching boundaries between neighbourhoods. Finally, if the algorithm reaches the local optima in the second stage – i.e., if the result is not improved after $g2$ iterations – the algorithm will stop.

2.2.4. Input data

We first utilise historical data of students' enrollment at the University of Alabama from 1971 to 1991 as the benchmark in the literature. Then, the quarterly Inter-bank foreign exchange market (IFEM) rates of Nigerian Naira against United States Dollar over the period January, 2004 to June, 2020 in accordance with (Pal and Kar, 2019).

Journal of Statistical Sciences & Data Analytics (JOSSDA)
Volume 1, Issue 2, December 2026 | Print ISSN XXXX-XXXX | Online ISSN XXXX-XXXX

DOI: 10.XXXXX/jossda.2026.1.1.015

2.2.5. Algorithm's parameters

The population size selected is problem-dependent, with the sizes of 20-50 as the most common sizes in the literature, and initial fireflies are generated randomly. The specific parameter values used for our algorithm are provided as follows: firefly swarm size: 30; total number of iterations: 200. Experiments are run 30 times. These values are selected based on exhaustive experimental trials conducted during the development of the algorithm. These parameters clearly influenced the computational cost of the proposed algorithm. To evaluate the efficiency of the forecasting model, MAD is used and calculated as follows:

$$MAD = \frac{\sum_{t=1}^{n} |Fv_t - d_t|}{n} \tag{11}$$

3. Results and Discussions

In this section, the computational results of the application of the proposed FA algorithm to the time-variant fuzzy time series are presented. The FA algorithm was run on a Pentium® Dual-Core @ 2.10GHz with 3.00 memory using the Python programming language. In order to demonstrate the efficiency and the effectiveness of the proposed algorithm, it is calibrated on a benchmark instance mentioned in the input data section. This instance has already been solved in the literature, and its optimality results can be used to compare algorithms.

3.1. Enrollments of the University of Alabama

The proposed algorithm is tested on a benchmark instance available in the literature. A comparison between the proposed algorithm and the particle swarm optimisation metaheuristic approach is first carried out. Then, the proposed algorithm is calibrated on the data in Table 3 to determine the length of the interval of discourse and its associated degree of membership values. The results of the proposed algorithm are presented in Table 1. We adopted MAD as a comparison metric to validate the performance of our proposed algorithm.

It is necessary to analyse the objective value of different values of *m* and *w* and select the best results for comparing with the PSO. Table 1 shows the results obtained from the proposed algorithm for different values of intervals (*m*) and window base (*w*). It can be observed from Table 1 that the algorithm achieves results that outperform the PSO approach at most of the values of (m, w). This can be attributed to the ability of the proposed algorithm to search for the solution efficiently through the embedded neighbourhood structure. In addition, the trends of combinations indicate that, at first, an increasing trend is observed in the objective values as the window base is increased; however, in the last few series, a decreasing trend is observed. Further, no special trend is observed along the number of intervals; therefore, the best is the combination of m = 14 and w = 8 (see the bold and asterisked value in column 8). Above all, the MAD for the proposed FA is lower than that achieved by PSO (see the highlighted columns).

Table 1. Comparison results between PSO and the proposed FA for Residual values of *m* and *w* for enrollments of the University of Alabama

m	$w = 2$	$w = 3$	$w = 4$	$w = 5$	$w = 6$	$w = 7$	$w = 8$
PSO							
5	419.3	390.2	390.72	396.91	417.16	422.61	357.42
6	402.2	383.9	379.03	394.66	414.81	421.94	353.01
7	406.7	378.9	385.06	387.76	410.29	424.08	**337.29**
8	407.5	369.7	375.87	381.77	406.13	412.34	352.67
9	403.7	383.9	395.01	376.41	403.77	418.53	**343.55**
10	410.3	373.8	356.42	374.46	405.23	412.50	**346.94**
11	400.9	374.1	375.12	370.74	405.45	417.11	**343.84**
12	401.1	377.3	378.70	371.52	404.86	415.63	**350.84**
13	411.5	369.9	360.19	373.49	396.69	412.78	**344.25**
14	394.4	373.2	387.56	361.07	403.67	413.10	**323.93**
Proposed FA							
5	411.5	389.3	389.6	392.56	415.56	420.67	**351.72**
6	400.1	381.5	375.6	391.34	414.23	421.22	**352.45**
7	403.4	378.2	384.2	384.25	410.15	422.51	**331.42**
8	403.0	368.8	372.2	380.36	405.65	410.65	**341.45**
9	402.0	382.6	390.25	375.82	401.45	415.38	**340.56**
10	402.1	373.5	355.12	374.12	405.31	412.22	**342.51**
11	400.2	372.4	372.15	370.58	404.26	415.58	**340.82**
12	401.3	375.3	377.82	370.45	402.75	412.53	**348.31**
13	410.5	368.2	360.21	373.26	392.56	410.81	**342.64**
14	392.8	370.8	386.45	360.38	401.82	412.42	**320.25***

Journal of Statistical Sciences & Data Analytics (JOSSDA)
Volume 1, Issue 2, December 2026 | Print ISSN XXXX-XXXX | Online ISSN XXXX-XXXX

DOI: 10.XXXXX/jossda.2026.1.1.015

Table 2. The comparison of proposed FA to PSO for Enrollments of the University of Alabama

No.	year	PSO							Proposed FA	
		$w = 2$	$w = 3$	$w = 4$	$w = 5$	$w = 6$	$w = 7$	$w = 8$	$w = 7$	$w = 8$
1	1971	-	-	-	-	-	-	-	-	-
2	1972	-	-	-	-	-	-	-	-	-
3	1973	-	-	-	-	-	-	-	-	-
4	1974	14160	-	-	-	-	-	-	-	-
5	1975	14696	15290	-	-	-	-	-	-	-
6	1976	15460	15859	15549	-	-	-	-	-	-
7	1977	15480	15538	15400	15381	-	-	-	-	-
8	1978	15772	15680	15692	15820	15872	-	-	-	-
9	1979	16154	15894	15950	16078	16130	16178	-	16123	-
10	1980	16807	17119	16807	16807	16807	17472	17534	16752	16375
11	1981	16919	16952	17008	16805	16884	16864	16668	16245	16210
12	1982	16388	16161	16234	16198	16175	16087	16137	16020	15965
13	1983	15433	15433	15433	15433	15433	15433	15433	15381	15381
14	1984	15497	15270	15544	15378	15429	15394	15246	15135	15234
15	1985	15145	15178	15192	15026	15051	15042	14894	15100	14565
16	1986	15163	15196	15252	15044	15069	15026	14912	15015	14682
17	1987	15984	16296	16386	15984	16168	16227	16711	15895	16235
18	1988	17509	17545	17522	17590	17283	17524	17586	16537	17443
19	1989	18941	18836	18911	18881	18855	18976	18877	18562	18198
20	1990	19471	19564	19372	19275	19424	19238	19410	18976	18980
21	1991	19328	19555	19417	19328	19560	19645	19328	18780	19245
22	1992	19506	19414	19426	19407	19421	19282	19086	18591	19176
m		**14**	**8**	**10**	**14**	**13**	**8**	**14**	**8**	**14**
MAD		**394**	**370**	**356**	**361**	**397**	**412**	**324**	**364**	**302**

Furthermore, in terms of the MAD, it can be observed in Table 2 that the proposed algorithm achieves superior results compared to other results found by the PSO. In fact, the proposed algorithm produces the smallest values for the $w = 7$ and $w = 8$ (the turquoise colour is the PSO results, while the yellow colour is the proposed FA results). Additionally, the forecasted values in Table 2 (columns 8 to 11) are plotted in Figure 4.

By graphical analysis of the variation of the forecasts, after application to the benchmark data of Alabama University enrolment, it seems clear that the proposed algorithm is the best one to determine the parameters (w, m) as compared to the PSO, because our algorithm yields the lowest forecasted enrollments as shown on the graph, which implies that the proposed FA can more efficiently be utilized to handle the exchange rate issue based on the time variant fuzzy time series.

Journal of Statistical Sciences & Data Analytics (JOSSDA)
Volume 1, Issue 2, December 2026 | Print ISSN XXXX-XXXX | Online ISSN XXXX-XXXX
 DOI: 10.XXXXX/jossda.2026.1.1.015

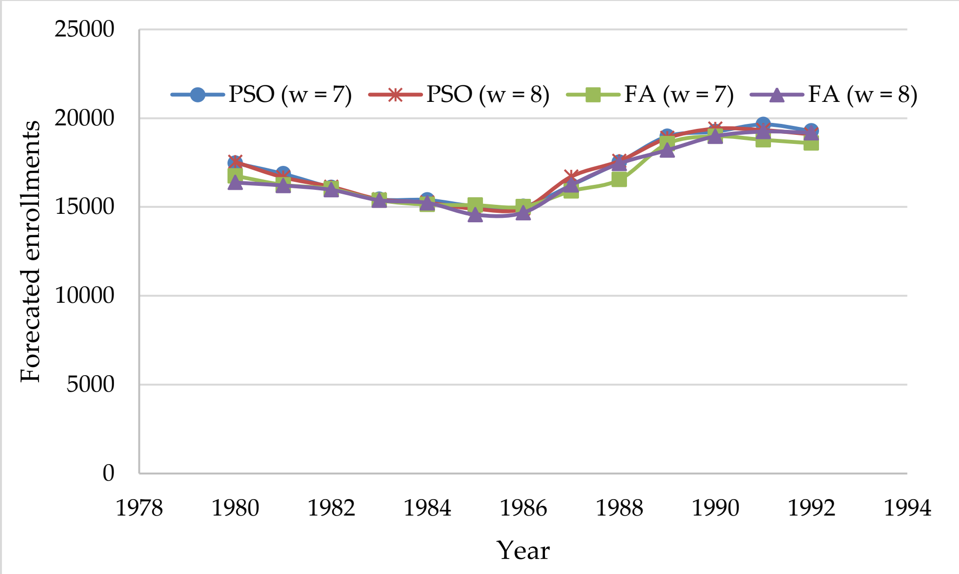

Figure 4. Plot of forecasted enrollments

3.2. Quarterly Inter-Bank Foreign Exchange Market

To validate the proposed FA, we calibrate it on the Inter-Bank Foreign Exchange Market (IFEM), rates of Nigerian Naira against the United States Dollar. The selected data includes 66 records of quarterly IFEM rates of NGN against USD derived from monthly average rates over the period January 2004 to June, 2020, which is shown in Table 3. All steps of the proposed method are followed similarly to the first example. Table 4 shows the results of the proposed algorithm for different values of intervals (m) and window base (w). In fact, the best result is obtained at w=8 and m=13, as shown in column 7.

Table 3. The historical data of IFEM rates of Nigeria

Year	Q1	Q2	Q3	Q4
2004	4.915	4.907	4.896	4.893
2005	4.891	4.895	4.902	4.873
2006	4.862	4.856	4.855	4.855
2007	4.855	4.850	4.840	4.791
2008	4.765	4.768	4.768	4.815
2009	4.591	4.308	5.029	5.013
2010	5.014	5.018	5.019	5.022
2011	5.034	5.046	5.036	5.077
2012	5.070	5.069	5.069	5.071
2013	5.061	5.067	5.084	5.070
2014	5.093	5.089	5.090	5.148
2015	5.253	5.283	5.283	5.283
2016	5.283	5.340	5.714	5.721
2017	5.722	5.723	5.723	5.723
2018	5.723	5.723	5.724	5.726
2019	5.726	5.727	5.727	5.727
2020	5.748	5.889		

Volume 1, Issue 2, December 2026 | Print ISSN XXXX-XXXX | Online ISSN XXXX-XXXX

DOI: 10.XXXXX/jossda.2026.1.1.015

Table 4. Residual values of *m* and *w* for IFEM rates of Nigeria

	Proposed FA						
m	$w = 2$	$w = 3$	$w = 4$	$w = 5$	$w = 6$	$w = 7$	$w = 8$
5	4.612	4.571	4.582	4.841	5.135	5.112	5.210
6	4.578	4.640	4.498	4.798	5.124	5.105	5.208
7	4.602	4.521	4.495	4.794	5.120	5.102	5.205
8	4.631	4.511	4.492	4.780	5.120	5.100	5.200
9	4.585	4.510	4.490	4.689	5.131	5.125	5.126
10	4.592	4.508	4.472	4.695	5.213	5.130	5.130
11	4.524	4.504	4.481	4.698	5.210	5.128	5.092
12	4.580	4.500	4.486	4.650	5.216	5.130	5.040
13	4.621	4.490	4.462	4.652	5.232	5.128	**5.026**
14	4.568	4.480	4.450	4.649	5.125	5.090	5.030

4. Conclusion

This paper presented a novel hybrid algorithm for the forecasting problem based on time-variant fuzzy time series and the firefly algorithm. The proposed algorithm determines the length of each interval in the universe of discourse and degree of membership values simultaneously. In order to analyse the number of intervals and window-based parameters and evaluate the efficiency of the proposed algorithm, benchmark data are selected and compared with the particle swarm optimisation approach in the literature based on forecasting accuracy; then, the proposed FA is applied to forecast the Inter-bank Foreign Exchange. The results indicate that the proposed algorithm performs satisfactorily and competes well with the Particle Swarm Optimisation approach.

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