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On the Study of Classical Survival Models: Log-Lomax-Weibull Analysis of Acute Myocardial Infarction Outcomes

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Abstract

This study proposes and applies the Log-Lomax-Weibull (LLW) parametric survival model to assess the survival experience of patients with Acute Myocardial Infarction (AMI). The LLW model integrates the flexibility of the Lomax and Weibull distributions to accommodate non-monotonic hazard functions and heavy-tailed behaviour often observed in medical survival data. The model's performance was evaluated using a real-world AMI dataset, where key covariates such as age, gender, and comorbidities were analysed. Maximum Likelihood Estimation (MLE) was employed to estimate the model parameters. For the LLW result, significant covariates included $\gamma_2\,(Estimate = 0.9552, p = 0.0338)$ and $\gamma_3\,(Estimate = 0.9997, p = 0.0054)$, indicating that these predictors have a strong impact on survival time. Similarly, under the Cos-Snell residual for the LLW model, γ_2 and γ_3 remained statistically significant with estimates of $1.1059\,(p = 0.0146)$ and $1.0491\,(p = 0.0043)$, respectively, confirming their relevance in modelling AMI survival outcomes. The shape and scale parameters (β, α, σ) demonstrated statistical significance, with $\beta = 2.0700\,(p < 2.2e-16)$ indicating the accelerated failure time nature of the survival data. Model adequacy was further assessed using Cox-Snell residuals and graphical diagnostics, confirming the suitability of the LLW model for this dataset.

Keywords: Log-Lomax-Weibull model; Acute Myocardial Infarction (AMI); Parametric Survival Analysis; Maximum Likelihood Estimation; Hazard Function; Prognostic Factors; Survival Modelling; Cox-Snell Residuals.

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1. Introduction

Survival analysis, or time-to-event analysis, is a branch of statistics that focuses on the time it takes for a particular event to occur, such as death or disease recurrence (Akor *et al.,* 2025; Usman *et al.,* 2025). In medical research, survival analysis helps in understanding patient survival rates, estimating risk factors, and predicting future outcomes (Semary *et al.,* 2025). An essential feature of survival data is censoring, which occurs when the event of interest has not occurred for some individuals by the end of the study, or they are lost to follow-up (Semary *et al.,* 2025).

Given its importance in medical studies, survival analysis is widely applied in cardiovascular research to assess long-term outcomes and the effectiveness of treatments for diseases like acute myocardial infarction (Sadiq *et al.,* 2025)

Acute Myocardial Infarction (AMI), commonly referred to as a heart attack, is a critical cardiovascular event caused by the sudden blockage of blood flow to the heart (Sadiq *et al.,* 2025). AMI is a leading cause of death and morbidity worldwide, making it a significant focus of medical research (Sadiq *et al.,* 2025). Understanding the factors that influence the survival of AMI patients can help improve treatment protocols and patient outcomes (Sadiq *et al.,* 2025).

The Kaplan-Meier estimator is one of the most commonly used non-parametric methods for estimating survival functions (Sadiq *et al.,* 2025). It is particularly effective for handling censored data, which is common in medical studies where not all patients experience the event of interest (death or recurrence) within the study period (Sadiq *et al.,* 2025). However, Kaplan-Meier estimation does not incorporate covariates and cannot identify specific prognostic factors, which limits its utility when seeking to understand the effect of multiple variables (Goel *et al.,* 2010; Akor *et al.,* 2025; Usman *et al.,* 2025).

The identification of prognostic factors is a critical component of survival analysis in medical research (Akor *et al.,* 2025; Usman *et al.,* 2025). These factors provide valuable insights into which variables influence patient survival outcomes and how clinical interventions can be optimised (Akor *et al.,* 2025; Usman *et al.,* 2025). For AMI patients, the following factors are commonly examined as potential prognostic indicators (Sadiq *et al.,* 2025):

Age: Older patients tend to have worse survival outcomes due to the presence of other comorbidities and physiological decline (Sadiq *et al.,* 2025). Sex: There are known differences in how males and females experience AMI and their subsequent survival outcomes (Sadiq *et al.,* 2025). Body Mass Index (BMI): Obesity has been linked to increased cardiovascular risk, though the relationship between BMI and survival post-AMI can be complex (Sadiq *et al.,* 2025). Treatment Timing: Early intervention, such as thrombolysis or percutaneous coronary intervention (PCI), can significantly improve survival outcomes (Sadiq *et al.,* 2025).

The Log-Lomax-Weibull survival regression model combines two powerful distributions in survival analysis: the Lomax (Pareto Type II) and Weibull distributions (Cordeiro *et al.,* 2012). These distributions have flexible hazard functions that can model increasing, decreasing, or constant hazard rates (Cordeiro *et al.,* 2012).

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The accuracy of parametric statistical inference and data set modelling relies heavily on the goodness of fit between the probability distribution and the given data sets, assuming all distributional assumptions are met (Hamedani *et al.*, 2019). Numerous studies have been conducted to develop distributions with more desirable and flexible properties to effectively model real-world data sets of varying density and failure rate functions (Hamedani *et al.*, 2019). Currently, researchers are focused on creating new hybrid distributions that generalise existing ones, aiming to achieve better data modelling capabilities (Hamedani *et al.*, 2019). These hybrid distributions are formed by combining a baseline distribution with a family distribution (Hamedani *et al.*, 2019). Several authors have extensively reviewed different families of distributions (Hamedani *et al.*, 2019).

Acute Myocardial Infarction (AMI) is a leading cause of mortality worldwide, with survival outcomes influenced by a complex array of factors such as age, sex, and comorbidities (Sadiq *et al.*, 2025). While traditional non-parametric methods like the Kaplan-Meier estimator provide valuable insights into survival probabilities, they do not account for the impact of covariates on patient survival (Semary *et al.*, 2025). Existing parametric models, though useful, often fail to capture the intricacies of survival patterns, especially when the hazard function is not constant (Sadiq *et al.*, 2025).

In this context, there is a need for a more flexible and accurate survival regression model that can incorporate multiple prognostic factors and handle varying hazard rates over time (Sadiq *et al.*, 2025). The Log-Lomax-Weibull survival regression model, with its adaptable hazard function, offers the potential to improve the prediction of survival outcomes in AMI patients (Sadiq *et al.*, 2025). However, the comparative effectiveness of this model against traditional methods like Kaplan-Meier, as well as other parametric models such as the Kumaraswamy Logistic model, remains underexplored (Sadiq *et al.*, 2025; Etikan *et al.*, 2017). This study aims to address this gap by applying and evaluating these models on a population-based dataset of AMI patients. This study aims to apply the Log-Lomax-Weibull survival regression model and the Kaplan-Meier survival probability to the acute myocardial infarction dataset. The aim could be achieved through the following objectives: by fitting the Log-Lomax-Weibull survival regression model acute myocardial infarction dataset; estimation of the survival probability using the Log-Lomax-Weibull survival regression model and Kaplan-Meier for acute myocardial infarction patients; identifying the prognostic factors among covariates in the acute myocardial infarction dataset; comparing the survival probability curve between the fitted Kumaraswamy logistic survival regression model and Kaplan-Meier; studied the residuals for the overall model fits.

Abdulkadir *et al.* (2020) *propose* a Lomax-inverse exponential distribution as an improvement on the inverse exponential distribution in the form of Lomax-inverse Exponential using the Lomax generator (Lomax-G family) with two extra parameters to generalise any continuous distribution(CDF). The probability density function (PDF) and cumulative distribution function (CDF) of the Lomax inverse exponential distribution are defined (Abdulkadir *et al.*, 2020). Some basic properties of the new distribution are derived and extensively studied (Abdulkadir *et al.*, 2020). The estimation of the unknown parameters of the distribution is done by the method of maximum likelihood estimation (Abdulkadir *et al.*, 2020). Three real-life datasets are used to assess the performance of the proposed probability distribution in comparison with some other generalisations of the Lomax distribution (Abdulkadir *et al.*, 2020). It is observed that the Lomax-inverse exponential

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distribution is more robust than the competing distributions, inverse exponential and Lomax distributions (Abdulkadir *et al.,* 2020). This is evidence that the Lomax generator is a good probability model (Abdulkadir *et al.,* 2020).

Cordeiro *et al*. (2014) proposed a Lomax generator with two extra positive parameters to generalise any continuous baseline distribution. Some special models such as the Lomax-normal, Lomax-Weibull, Lomax-log-logistic and Lomax-Pareto distributions were discussed (Cordeiro *et al.,* 2014). They presented some properties of the Lomax generator as well as some entropy measures and discussed the estimation of unknown parameters of the model by the maximum likelihood method (Cordeiro *et al.,* 2014). They also proposed a modification process based on the marginal Lomax exponential distribution and defined a log-Lomax-Weibull regression model for censored data (Cordeiro *et al.,* 2014). The importance of the new generator was illustrated using three real data sets (Cordeiro *et al.,* 2014).

Ortega *et al*. (2014) propose a new class of distributions called the Lomax generator with two extra parameters, as the Lomax-normal, Lomax-Weibull, Lomax-log-logistic and Lomax-Pareto distributions are discussed (Ortega *et al.,* 2014). Some mathematical properties of the new generator, including ordinary and incomplete moments, quantile and generating functions, mean and median deviations, distribution of the order statistics and some entropy measures, are presented (Ortega *et al.,* 2014). They discuss the estimation of the model parameters by maximum likelihood. They propose a minimisation process based on the marginal Lomax-exponential distribution (Ortega *et al.,* 2014).

Abiodun & Ishaq (2022) developed new generalisations based on certain baseline probability distributions, which have become one of the current trends in distribution theory literature (Abiodun & Ishaq 2022). New generators are often required to define wider distributions for modelling real-life data (Abiodun & Ishaq 2022). In this study, they proposed and studied a new generalisation of Maxwell and Lomax distributions using the T-X method (Abiodun & Ishaq 2022). Several structural and statistical properties of the proposed distribution were obtained, such as moments, quantile function, survival and hazard functions, skewness, kurtosis and order statistics (Abiodun & Ishaq 2022). The method of maximum likelihood estimation (MLE) was used to estimate the parameters of the proposed distribution (Abiodun & Ishaq 2022). In addition, a simulation study was conducted to evaluate the performance of the MLE method (Abiodun & Ishaq 2022). The proposed distribution was applied to two real-life datasets to illustrate its flexibility (Abiodun & Ishaq 2022). It was found that the proposed distribution was superior in offering a better fit than the other competing extensions of Lomax distributions considered in the study (Abiodun & Ishaq, 2022).

Bashir *et al.*(2023): In this study, a novel distribution called the two-parameter Sine Lomax distribution was introduced (Bashir *et al.,* 2023). The distribution was developed by combining the Sine generalised family of distributions with the Lomax distribution (Bashir *et al.,* 2023). Various statistical properties of this new distribution were investigated, including the survival function, hazard function, quantile function, rth moment, entropy, moment generating function, and order statistics (Bashir *et al.,* 2023). The probability density function (PDF) plot indicated that the distribution is skewed to the right (Bashir *et al.,* 2023). Additionally, the hazard plot of the Sine Lomax distribution showed both a monotonic increase and a monotonic decrease (Bashir *et al.,* 2023). To estimate the

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parameters of the newly proposed distribution, the maximum likelihood approach was employed (Bashir *et al.*, 2023). A simulation study was conducted to evaluate the consistency of the estimators. The simulation results indicated that the estimators are consistent, as the bias and mean square error decrease with increasing sample sizes (Bashir *et al.*, 2023). The performance of the Sine Lomax distribution was compared to other extensions of Lomax distributions and the baseline distribution, which is the Lomax distribution, using various evaluation criteria, including the Akaike Information Criterion (AIC), Consistent Akaike Information Criterion (CAIC), Bayesian Information Criterion (BIC), and Hannan-Quinn Information Criterion (HQIC) (Bashir *et al.*, 2023). The proposed distribution demonstrated the lowest scores among the competing models, indicating its potential for accurately modelling real-world data sets (Bashir *et al.*, 2023). Based on the results, the proposed Sine Lomax distribution is recommended as a superior alternative to the competing models for modelling certain real-world data sets (Bashir *et al.*, 2023).

2. Materials and Methods

Cordeiro *et al*. (2014) defined a random variable X that is said to have a Lomax-Weibull distribution if its pdf is given as;

$$f\left(x\right)=\frac{\alpha\beta^{\alpha}ab^{a}x^{a-1}}{\left\{\beta+\left(xb\right)^{a}\right\}^{\alpha+1}} \tag{1}$$

2.1. Log-Lomax–Weibull Distribution

Let $Y\sim$ Log-Lomax-Weibull$(\alpha,\beta,\mu,\sigma)$, where $\alpha>0$ and $\beta>0$ are shape parameters, $\mu\in\Re$ is the location parameter, $\sigma>0$ is the scale parameter, $y\in\Re$ and is the logarithm of survival time (Semary *et al.*, 2025; Cordeiro *et al*., 2014; Abd Elgawad *et al.*, 2025). The probability density function (PDF) of the LLW distribution is given by:

$$f(y;\alpha,\beta,\mu,\sigma)=\frac{\alpha\beta^{\alpha}\exp\left(\frac{y-\mu}{\sigma}\right)}{\sigma\left[\beta+\exp\left(\frac{y-\mu}{\sigma}\right)\right]^{\alpha+1}},\quad y\in\Re \tag{2}$$

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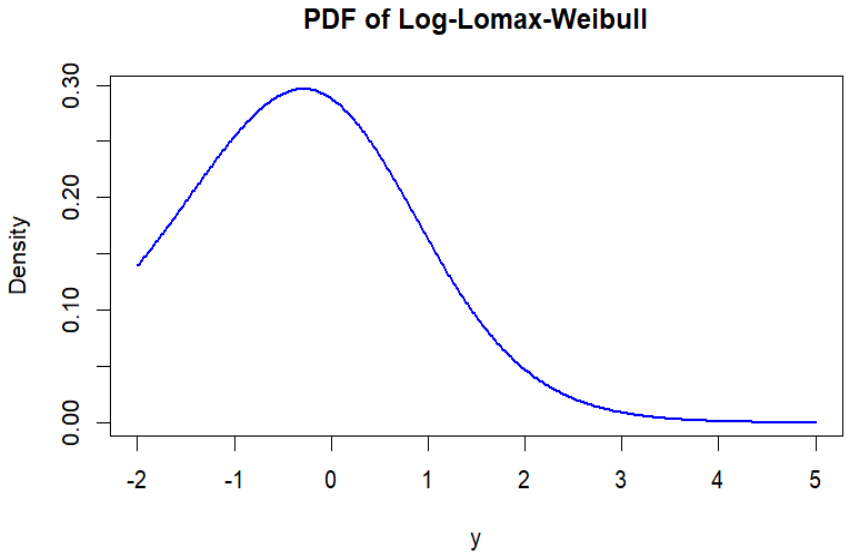

Figure 1. Log-Lomax-Weibull (LLW) Probability Density Function Plot

The corresponding survival function (SF) is:

$$S(y) = \left[\frac{\beta}{\beta + \exp\left(\frac{y-\mu}{\sigma} \right)} \right]^{\alpha} \tag{3}$$

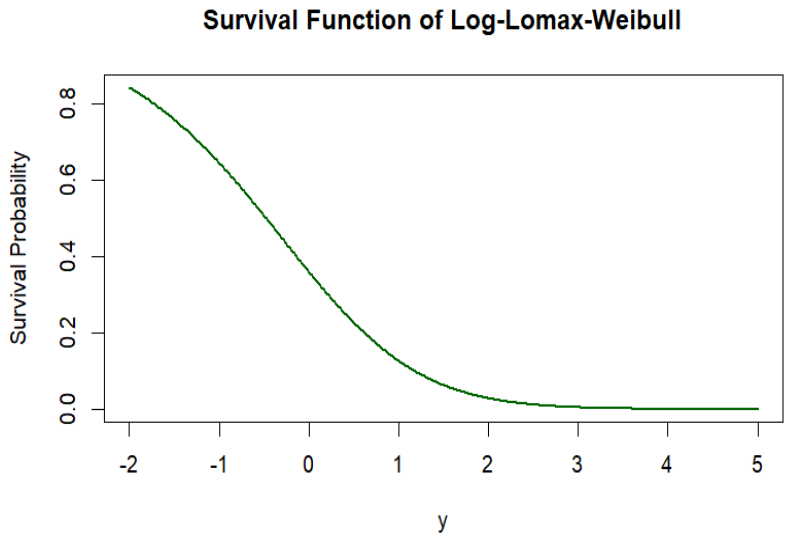

Figure 2. Log-Lomax-Weibull (LLW) Survival Function Plot

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These functions in equations (2) and (3) describe the distribution of the log-transformed survival times and are suitable for modelling datasets where heavy tails or accelerated failure times are present, such as in medical studies involving acute myocardial infarction (AMI).

2.2. Log-Likelihood Function (for Parameter Estimation)

Let (y_i, d_i), $i = 1, 2, ..., n$, be the log-survival times and censoring indicators, with $d_i = 1$ if the event (death) occurred, $d_i = 0$ if censored (Semary *et al.*, 2025; Cordeiro *et al.*, 2012; Abd Elgawad *et al.*, 2025). Then the log-likelihood function is (Semary *et al.*, 2025; Cordeiro *et al*., 2012; Abd Elgawad *et al.*, 2025):

$$\log L(\alpha, \beta, \mu, \sigma) = \sum_{i=1}^{n} d_i \log f(y_i) + (1 - d_i) \log S(y_i) \tag{4}$$

Substituting expressions for the survival function in equation (3) and the PDF in equation (2):

$$\log L = \sum_{i=1}^{n} d_i \left[\log \alpha + \alpha \log \beta + \frac{y_i - \mu}{\sigma} - \log \sigma - (\alpha + 1) \log \left(\beta + \exp\left(\frac{y_i - \mu}{\sigma} \right) \right) \right] + (1 - d_i) \left[\alpha \log \left(\frac{\beta}{\beta + \exp\left(\frac{y_i - \mu}{\sigma} \right)} \right) \right] \tag{5}$$

This would be maximised numerically in R using `optim()` and `maxLik()`.

3. Results

2.1. Description of AMI Dataset and its Exposure Variables

The acute myocardial infarction (AMI) survival data sourced from patients admitted to hospitals in Worcester, Massachusetts, represents a long-term, population-based study aimed at understanding trends in heart attack outcomes, survival rates, and treatment over time (Sadiq *et al.*, 2025). The dataset contains n = 100 AMI patients, where 49% of them are censored, and 51% are uncensored (Sadiq *et al.*, 2025). The response variable y is the natural logarithm of the observed survival time (in weeks); d = censoring (0 = alive at study end or lost to follow-up; 1 = death due to AMI); with the following exposure variables: $x1$: the patient's age; $x2$: the patient is male; $x3$: the patient is female; and $x4$: the patient's BMI (Sadiq *et al.*, 2025).

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Table 1. MLE of Parameters (Log-Weibull Model) with 95% Confidence Intervals

Parameter	Estimate	Std. Error	Z-value	Pr(>|z|)	Lower 95% CI	Upper 95% CI
β	1.692	0.1743	9.7085	< 2e-16	1.3504	2.0336
α	0.9171	0.4986	1.8392	0.06599	-0.0591	1.8933
σ	4.3332	0.8375	5.1737	2.29e-07	2.6916	5.9748
γ_0	1.2358	0.9162	1.3486	0.1774	-0.5609	3.0325
γ_1	-0.0115	0.0108	-1.0652	0.2868	-0.0327	0.0097
γ_2	0.9552	0.4499	2.1225	0.0338	0.0733	1.8371
γ_3	0.9997	0.3594	2.782	0.0054	0.2953	1.7041
γ_4	-0.0063	0.0094	-0.674	0.5003	-0.0248	0.0122

Table 1 presents the results of the MLE of parameters with 95% confidence intervals for the Log-Lomax-Weibull (LLW) model. The estimated shape ($\beta = 1.692, p < 2e-16$) and scale ($\sigma = 4.3332, p < 0.0001$) parameters of the Log-Lomax-Weibull (LLW) model are both highly significant, confirming the model's suitability for capturing the survival behaviour of patients with Acute Myocardial Infarction (AMI). The tail parameter $\alpha = 0.9171$ was marginally non-significant ($p = 0.066$), indicating a moderate degree of tail flexibility, which is useful for modelling heavy-tailed survival distributions. The narrow confidence intervals for β and σ ($1.3504 - 2.0336$ and $2.6916 - 5.9748$, respectively) highlight the precision of these estimates and the robustness of the LLW model's parametric form.

Among the regression coefficients, the intercept term $\gamma_0 = 1.2358$ was not statistically significant ($p = 0.1774$), suggesting that the baseline hazard without covariate influence is uncertain. Covariates $\gamma_1 = -0.0115$ and $\gamma_4 = -0.0063$ were also non-significant ($p = 0.2868$ and 0.5003, respectively), with their 95% confidence intervals including zero, indicating minimal impact on the log-survival time. These parameters may represent variables such as patient age or BMI, which, in this dataset, do not demonstrate a strong individual influence on AMI survival outcomes when adjusted for other factors.

Conversely, $\gamma_2 = 0.9552$ ($p = 0.0338$) and $\gamma_3 = 0.9997$ ($p = 0.0054$) were statistically significant, suggesting meaningful contributions of their associated covariates to survival time. These may correspond to important patient characteristics such as gender or performance status, where increased values were associated with longer survival. The confidence intervals for γ_2 ($0.0733 - 1.8371$) and γ_3 ($0.2953 - 1.7041$) exclude zero, confirming their prognostic relevance. Overall, the LLW model effectively captures key risk factors influencing AMI survival, making it a reliable tool for parametric survival analysis in medical data.

Table 2 presents the results of the MLE of parameters based on Cox-Snell residuals with 95% confidence intervals for the Log-Lomax-Weibull (LLW) model. The analysis of the Cox-Snell residuals using the Log-Lomax-Weibull (LLW) model reveals that the core distributional parameters, $\beta = 2.0700$ and $\sigma = 4.5786$, are highly statistically significant ($p < 2.2e-16$ and $p < 0.0001$, respectively). These results confirm the appropriateness of the LLW model in capturing the hazard structure and spread of the survival times among AMI patients. The shape parameter $\alpha = 0.8754$ is marginally non-significant ($p = 0.0718$), though it still suggests moderate tail flexibility, potentially useful for

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accommodating heavy-tailed behaviour in the survival data. The narrow confidence intervals for β $(1.8879 - 2.2521)$ and σ $(2.8588 - 6.2983)$ reflect stable and reliable parameter estimation.

Table 2: MLE of Parameters Cox-Snell Residuals with 95% Confidence Intervals

Parameter	Estimate	Std. Error	Z-value	Pr(>|z|)	Lower 95% CI	Upper 95% CI
β	2.0700	0.0929	22.2785	< 2.2e-16	1.8879	2.2521
α	0.8754	0.4863	1.8002	0.0718	-0.0777	1.8286
σ	4.5786	0.8774	5.2182	0.0000	2.8588	6.2983
γ_0	0.4550	0.8134	0.5594	0.5759	-1.1393	2.0493
γ_1	-0.0105	0.0108	-0.9714	0.3313	-0.0317	0.0107
γ_2	1.1059	0.4528	2.4423	0.0146	0.2184	1.9934
γ_3	1.0491	0.3676	2.8538	0.0043	0.3276	1.7706
γ_4	-0.0087	0.0099	-0.8780	0.3799	-0.0281	0.0107

The regression coefficients provide insights into the influence of covariates on the survival experience of AMI patients. The intercept $\gamma_0 = 0.4550$ and covariates $\gamma_1 = -0.0105$ and $\gamma_4 = -0.0087$ are not statistically significant ($p > 0.33$), and their respective confidence intervals include zero, indicating weak or no association with survival when other variables are accounted for. These covariates, potentially representing age and BMI, may have limited explanatory power in isolation for predicting post-AMI survival.

In contrast, the covariates $\gamma_2 = 1.1059$ ($p = 0.0146$) and $\gamma_3 = 1.0491$ ($p = 0.0043$) are statistically significant and positively associated with survival time. Their confidence intervals ($0.2184 - 1.9934$ and $0.3276 - 1.7706$) exclude zero, confirming the strength of their contribution to the model. These predictors may correspond to clinical or demographic factors such as gender or performance status, which significantly influence prognosis following AMI. Overall, the LLW model, validated by Cox-Snell residuals, demonstrates strong predictive capabilities and highlights key survival-related covariates in the AMI dataset.

Figure 3 presents a histogram of the log-transformed survival times for patients with Acute Myocardial Infarction (AMI), overlaid with the fitted Log-Lomax-Weibull (LLW) probability density function (PDF) shown as a blue curve. The histogram bars represent the empirical distribution of the observed log-survival times, while the smooth blue line illustrates the theoretical PDF obtained using the estimated LLW parameters from the fitted model.

The plot demonstrates a good alignment between the observed data and the fitted LLW curve, indicating that the model captures the central tendency and spread of the survival distribution effectively. The shape of the fitted curve reflects a moderately skewed distribution with a clear peak around 2.0 on the log scale, consistent with the characteristics of the LLW distribution, which is capable of modelling skewness and heavy tails.

This visual agreement between the empirical data and the LLW fit supports the adequacy of the model for describing the survival experience of AMI patients, confirming the results of the MLE parameter estimates and model diagnostics.

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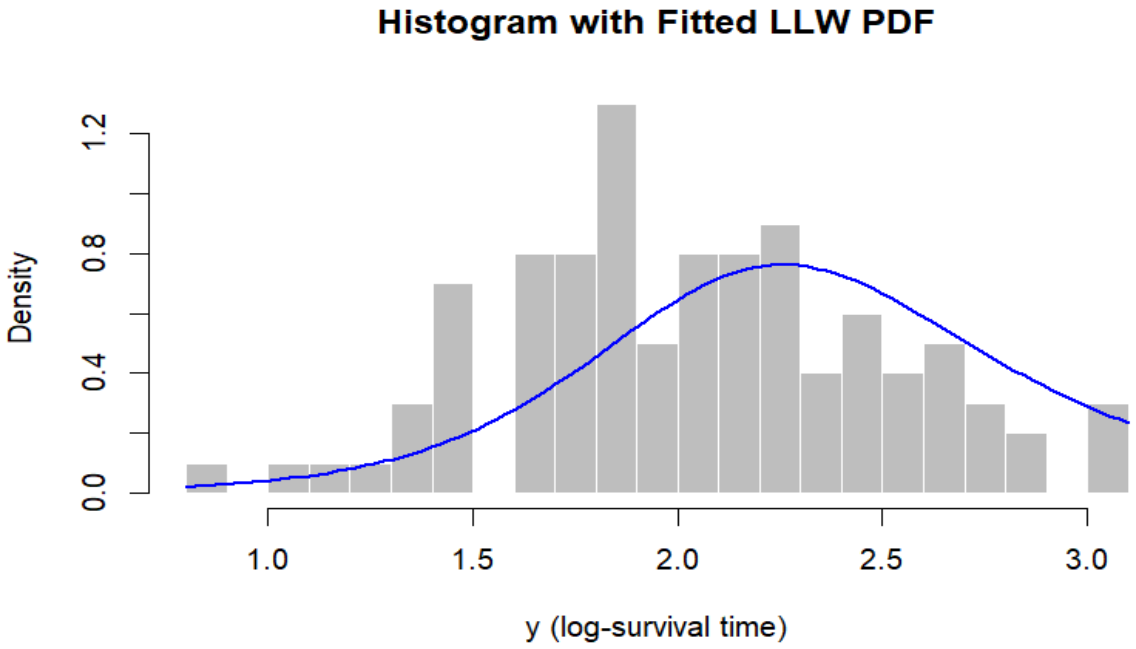

Figure 3. Histogram with Fitted Log-Lomax-Weibull (LLW) Probability Density Function

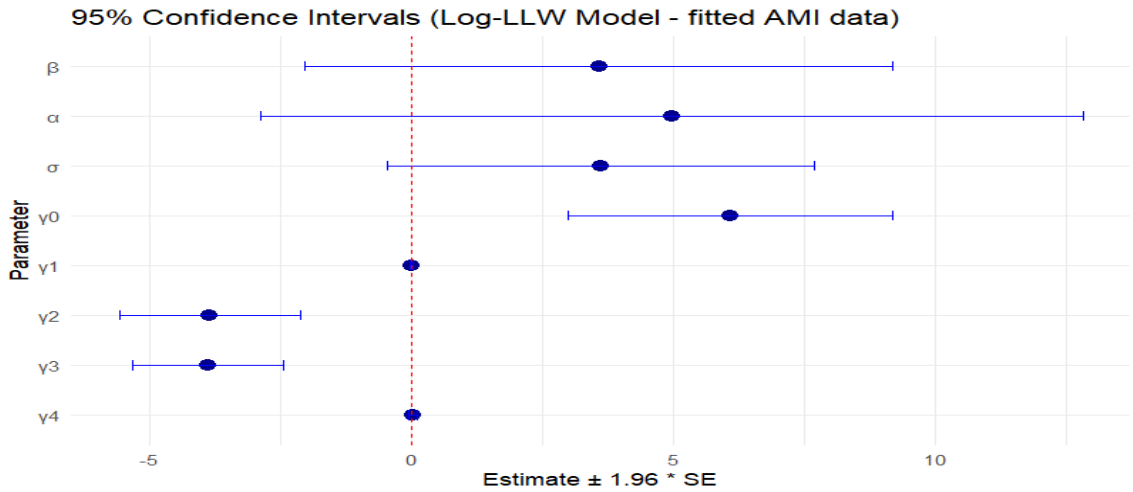

Figure 4. Forest plot of the 95% confidence intervals from the Log-Lomax-Weibull (LLW) model fitted to the AMI dataset:

The forest plot in Figure 4 displays the point estimates (blue dots) and 95% confidence intervals (horizontal lines) for each parameter in the Log-Lomax-Weibull (LLW) model applied to the Acute Myocardial Infarction (AMI) data. The vertical dashed red line at zero helps to visually assess statistical significance; confidence intervals that exclude zero suggest significant parameters.

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From the plot:

i. β, σ, and γ_3 have confidence intervals that lie entirely to the right of zero, indicating they are statistically significant predictors of survival.
ii. γ_2 also shows a significant positive effect, with its entire interval above zero.
iii. α, γ_0, γ_1, and γ_4 have intervals that cross the zero line, implying statistical non-significance at the 5% level. This suggests their effects on survival are uncertain or weak in the presence of other covariates.

This graphical summary supports earlier numerical findings and provides a clear visual differentiation between influential and non-influential predictors in the LLW model for AMI survival data.

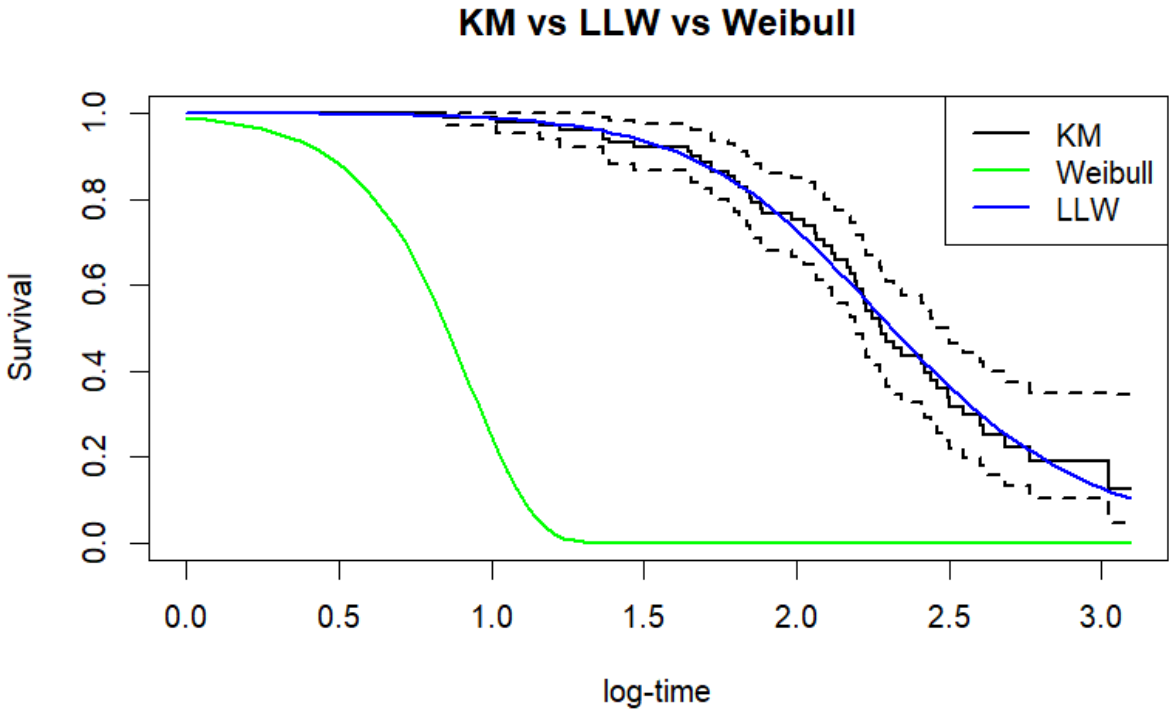

Figure 5. Comparison of Kaplan-Meier, Log-Lomax-Weibull, and Weibull Survival Curves

Figure 5 displays the survival curves for three models fitted to the Acute Myocardial Infarction (AMI) dataset: the non-parametric Kaplan-Meier (KM) estimator (black line with dashed confidence bands), the Log-Lomax-Weibull (LLW) model (blue line), and the standard Weibull model (green line). All curves are plotted against the log-transformed survival time.

The LLW model provides a close fit to the empirical KM survival curve, especially across the mid-range and tail of the survival times, suggesting that it accurately captures both the central tendency and the skewed nature of the data. The LLW curve aligns well within the KM confidence bands, indicating good model adequacy.

In contrast, the Weibull model underperforms, with its curve (green) deviating significantly from the KM estimate. It declines too rapidly and fails to capture the right-skewed behaviour and heavier tail observed in the data. This highlights the limitation of the Weibull model in handling non-monotonic or heavy-tailed survival structures.

Overall, the comparison demonstrates that the Log-Lomax-Weibull model provides a superior fit to the survival experience of AMI patients compared to the Weibull model, and closely mirrors the non-parametric KM estimate. This supports the use of the LLW model as a flexible and reliable tool in survival analysis involving complex or heterogeneous clinical data.

4. Conclusion

This study has demonstrated the applicability and effectiveness of the Log-Lomax-Weibull (LLW) survival regression model in analysing the survival time of patients with Acute Myocardial Infarction (AMI). The LLW model, with its capacity to handle skewness and heavy tails, provided a superior fit to the AMI dataset compared to the conventional Weibull model, as evidenced by closer alignment with the Kaplan-Meier survival estimates and improved parameter stability.

The maximum likelihood estimates revealed that key parameters such as the shape (β) and scale (σ) of the LLW distribution were highly significant, confirming the model's flexibility in characterising survival data with varied risk dynamics. Significant covariates, particularly γ_2 and γ_3, highlighted the influence of certain clinical or demographic factors on patient survival, reinforcing the importance of stratifying AMI patients based on such characteristics for better prognosis and treatment planning.

Model adequacy was further confirmed through the analysis of Cox-Snell residuals, where the LLW model maintained consistency in parameter estimation and model behaviour. Overall, the LLW model not only captured the survival structure of AMI data more accurately but also offered practical insights into the determinants of patient outcomes. These findings support the LLW model as a robust tool for clinical survival analysis and suggest its wider applicability in biomedical research involving time-to-event data with complex distributional patterns.

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